#### LEXINGTON SCHOOL COMMITTEE MEETING

#### Tuesday, January 13, 2015 Lexington Town Office Building, Selectmen's Meeting Room 1625 Massachusetts Avenue

#### 7:30 p.m. <u>Call to Order and Welcome</u>:

Public Comment – (Written comments to be presented to the School Committee; oral presentations not to exceed three minutes.)

#### 7:35 p.m. <u>Superintendent's Announcements</u>:

#### 7:45 p.m. <u>School Committee Member Announcements:</u>

#### **7:55 p.m.** Agenda:

1. Discussion of the Recommendation from the Ad hoc School Master Planning Committee (60 minutes)

## 8:55 p.m. Public Hearing on the Recommendation of the Ad hoc School Master Planning Committee (30 minutes)

#### **9:25 p.m. Agenda** (continued):

- 2. Discussion of Filing a Statement of Interest with the Massachusetts School Building Authority for Hastings School (15 minutes)
- 3. Presentation of the Report from the Enrollment Working Group (20 minutes)

#### 10:00 p.m. Adjourn:

#### The next scheduled meetings of the School Committee are as follows:

- Tuesday, January 20, 2015 7:30 p.m., Town Offices Building, Selectmen's Meeting Room, 1625 Massachusetts Avenue
- Tuesday, January 27, 2015 Public Hearing on the Superintendent's FY 16 Recommended Budget –
   7:30 p.m., Town Offices Building, Selectmen's Meeting Room, 1625 Massachusetts Avenue

All agenda items and the order of items are approximate and subject to change.

# **Ad hoc School Master Planning Committee**

# **Option 9**



Recommended January 8, 2015



# **Recommendation (page 1)**

For September of 2016, construct additional educational space (prefabricated modular construction) at Bridge, Bowman, Clarke and Diamond to alleviate current overcrowding and create swing space for a Diamond addition.

**Total Project Cost** 

Bowman School \$3,100,000

Bridge School \$3,680,000

- Two classrooms, one Music room, each

Clarke School \$4,610,000

Five classrooms

#### **Diamond School**

- Eight Classrooms \$4,500,000 (remove four modular classrooms) \$15,890,000

January 8, 2015 2



# **Recommendation (page 2)**

For September of 2017, construct additional educational space (brick and mortar construction) at Harrington for elementary and pre-k students and a new addition at Diamond for middle school students.

**Total Project Cost** 

### Harrington

\$24,300,000

- Expanded pre-k, six classrooms, music, art and special education spaces

#### **Diamond**

- 15 classrooms in new wing

\$18,420,000

\$42,720,000

January 8, 2015



# **Recommendation (page 3)**

Advance new Hastings project.

Without MSBA grant, available September 2018

Total Project Cost \$59,000,000

With MSBA grant, earliest available September 2020 Lexington cost 68% of total \$40,100,000

January 8, 2015 4



## Total cost\* of Option 9 recommendation:

Without MSBA grant

\$117,610,000

With MSBA grant

\$98,710,000

\*Costs are preliminary, for planning purposes, and will change as plans and designs are developed.

January 8, 2015 5

#### 12/17/2014

Lexington School Committee Lexington Public Schools 146 Maple Street Lexington, MA 20420 LPS Enrollment Working Group

Dear Madam Chairman and School Committee Members,

The Enrollment Working Group is pleased to let you know that we have completed our work to understand the sources of the Lexington Public School enrollment growth that has been ongoing for the last several years, and has produced our final forecasts of the expected enrollment growth over the next five years.

Attached is our final report which describes these sources of enrollment growth, the data and methodology we have employed to analyze the past growth and forecast the growth for the next five years, and the final forecasts.

We appreciate the opportunity to perform this work for the town of Lexington and the school system and we hope these analyses and forecasts will be of benefit to the School Committee.

Sincerely,

Mark Andersen

**Rodney Cole** 

Tim Dunn

Dan Krupka

Joe Pato

# Five-year Enrollment Forecasts for the Lexington Public School:

FY2015-FY2020 Report of the Enrollment Working Group

#### TABLE OF CONTENTS

1	E	xecutive Summary	.2
2	li	ntroduction	.5
3	F	Historical Review	.6
4		Data Sources	.9
5	[	Data analysis	LO
	5.1	Analysis of housing stock	LO
	5.2	Percentage of housing units occupied by families with students	L1
	5.3	Average number of students per housing unit where students are present	L2
	5.4	Enrollment volatility	L3
	5.5	Churn in enrollment	L3
6	F	Forecasting methods	L4
	6.1	Cohort Survival Method	L4
	6.2	Linear Extrapolation of enrollment	L5
	6.3	Housing Demographic Method	L5
7	F	ive-Year Forecasts	L6
	7.1	Forecasts for the high school and the middle schools	L6
	7.2	Forecast for the elementary schools	L8
	7.3	Summary of forecasts	20
8		Discussion	21
9	F	Recommended next steps	22
1(	)	Acknowledgments	22
Αį	ppen	dices	23
	App	pendix A - Ex-post forecasts	23
	App	pendix B - Estimation of confidence intervals	27
	App	pendix C – Data used to construct the forecasts and most of the figures	29
	F	Historical values of enrollment and births (LPS data)2	29
		Data on housing by category	29
	P	Percentage of housing units occupied by students (system-wide)2	29
	P	Percentage of housing units by elementary-school students	30
		Average number of students in a housing unit when at least one student is present – System-wide student density	30
	A	Average number of elementary-school students in a housing unit when at least one elementary school student is present – Elementary-school student density	
	3	reactives present. Elementary-school stadent density	,0

#### **1** EXECUTIVE SUMMARY

Enrollment in the Lexington Public Schools (LPS) has risen by 607 students or 10 percent between the 2008-2009 and the 2014-2015 school years. Over this period, the number of students in the elementary grades grew by 347, the equivalent of about two thirds of an elementary school of 550. The growth had not been anticipated: as late as January 2011, the forecast for elementary school enrollment had projected a slight decline over the following five years. As enrollment continued to increase, the Superintendent, Dr. Paul Ash, concluded that a better forecasting method was needed, and decided to engage a consultant. To assist him with the selection process he formed, in November 2013, an advisory group of five residents with relevant expertise, experience and interest. After reviewing the proposals submitted by the responding consultants, the members of the advisory group concluded that the task was more complex and challenging than acknowledged by the consultants, and offered to take on the task themselves.

Thus was born the Enrollment Working Group (EWG). Its charter is to develop five-year enrollment forecasts for the elementary grades (K-5), the middle school grades (6-8) and for the high-school grades (9-12). As the EWG assessed past LPS forecasts, it concluded that the method used in Lexington and in most other communities, the so-called Cohort Survival Method (CSM), had a major shortcoming in developing an estimate of future Kindergarten enrollment. The CSM uses historical grade-by-grade enrollment data to calculate the ratio of students in a given grade to the number of students in the preceding grade during the previous school year. These so-called progression rates are usually averaged over five years and typically range in value from 1.0 to 1.1. Using current enrollment as the starting point, they are applied to forecast future grade-by-grade enrollment. The EWG found that, while the CSM's historical forecasts for the middle schools and the high school were adequate, the projections for the elementary grades consistently underestimated enrollment owing to the method of projecting future Kindergarten enrollment. Traditionally, its value in a given year was estimated as the product of the births to Lexington families five years earlier and the so-called Birth-to-Kindergarten progression rate. This method is appropriate when births are a good predictor for the number of students entering Kindergarten five years later. For the past ten years, however, there has been virtually no correlation between Kindergarten enrollment and births (five years earlier) because of large net in-migration of families with pre-school children. Meanwhile, the Birth-to-Kindergarten progression rate reached a value of 2.4 in FY2013, up from 1.4 ten years earlier. In effect, for each child born to a Lexington mother more than one additional child enrolled in Kindergarten five years later.

In light of these trends, the EWG concluded that it would have to achieve a fundamental understanding of the factors controlling enrollment. Accordingly, it assembled and analyzed data on the following three variables:

- 1. The number of housing units. The total number of units is about 11,300 now. Single-family homes comprise about 9,100, condominiums and apartments about 1,000 each and multi-family units about 200.
- 2. The percentage of the units occupied by families with students in the LPS. This is approximately 36 percent.
- 3. The average number of students in a housing unit with at least one student. This is now about 1.6.

Using these data, the EWG found that virtually all the increase in enrollment from 2003 to 2013 could be attributed to growth in the number of students living in apartments. The finding was reported in the EWG's first presentation to the School Committee on March 11, 2014. The EWG then continued to analyze the relationship between historical values of enrollment and the three variables. In one study the EWG examined trends in the percentage of units occupied by families with students – by category of housing, e.g., single-family homes, apartments and condominiums. The analysis revealed that there remains substantial headroom for this variable in apartments: By 2020 it is projected to increase to about 56 percent. For single-family homes the corresponding value is expected to be about 38 percent.

The EWG also assessed the strengths and shortcomings of three forecasting methods:

- The Modified Cohort Survival Method (MCSM) a version of the CSM in which future
  Kindergarten enrollment is assumed to be equal to the average of Kindergarten enrollment in
  recent years.
- 2. The Linear Extrapolation Method (LEM) a linear extrapolation of the recent enrollment
- 3. The Housing Demographic Method (HDM) a forecast based on the product of extrapolations of the three housing demographic variables

The assessment consisted of evaluating the accuracy of each method if it had been used in the past. The EWG found that LEM produced the largest forecast errors, particularly in the vicinity of turning points — years when enrollment stopped decreasing and began to increase. Although the MCSM lacked the capability to forecast accurately in the vicinity of turning points, it performed somewhat better than the LEM. By contrast, the HDM showed promise of being able to forecast turning points owing to the forecasters' ability to anticipate changes in the number of housing units and the percentage of housing units occupied by students.

On the basis of these assessments, the EWG decided to use the CSM for five-year forecasts for the middle schools and the high school because its historical forecast accuracy was judged adequate and because it is so simple to use. (Since historical Kindergarten enrollment is not required for five-year forecasts of middle school enrollment, CSM and MCSM give the same results). For forecasting elementary school enrollment, the EWG selected the HDM on the basis of its potential to anticipate turning points.

The table below presents the EWG's forecasts for enrollment in FY2020, growth relative to FY2014 and the associated 90 percent confidence limits. Because the confidence intervals for the two methods are calculated using different methods, they are not directly comparable. The width of the confidence interval for elementary grades lends support to the decision to limit the horizon for enrollment forecasts to five years.

Grade Group	Method	Enrollment in FY2020	Growth over FY2014
Elementary (K-5)	HDM	3188 ± 267	260 ± 267
Middle School (6-8)	CSM	1830 ± 70	171 ± 70
High School (9-12)	CSM	2290 ± 120	269 ± 120
Total System	HDM	7279 ± 410	671 ± 410

For planning facilities, it is essential to take into account the confidence limits. Thus, if elementary school enrollment growth were to reach the upper limit of 527 in FY2020, it would represent the

addition of the equivalent of a Lexington elementary school. It is also worth noting that the enrollment projected for the total system in FY2020 is still about 2,500 students lower than the all-time peak in 1969.

Our forecasts do not include any consideration of changes in the national or local economies that would have an impact on the projected growth. For the next several years, the EWG believes there is enough headroom in existing housing units to accommodate further growth in enrollment. It remains to be seen whether the percentage of apartment units occupied by students will continue to grow at its current rate. In part, this will be determined by how rapidly the apartments turn over. In light of the importance of this factor, as well as the turnover rates of the other housing categories, the EWG has started to collect and analyze the relevant data. In addition, the EWG plans to help members of the LPS administration to assume responsibility for performing future forecasts.

#### 2 Introduction

The November 2006 enrollment report issued by the LPS projected that enrollment in the elementary schools would drop by more than 400 over the following five years. A school closing loomed as a possibility. Yet, in 2011, five years after the forecast, elementary school enrollment had risen by more than 150 students. Now, in December 2014, four of Lexington's six elementary schools are considered to be at or over capacity. Many explanations can be offered for this surprise: the growing reputation of our schools, economic recovery in the Boston metropolitan area, burgeoning biotech and high-tech environments, and a Town population becoming progressively more diverse.

About one year earlier, in November 2013 Dr. Paul Ash, Superintendent of Lexington Public Schools, decided to engage a consultant to develop a more reliable method of forecasting enrollment. To assist him, he formed an advisory group to draft the requisition for proposals and to select the winner. After reviewing the proposals, the members of advisory group concluded that the proposals failed to recognize the magnitude of the challenge, and offered to take on the task themselves. And thus, in January 2014, the advisory group was transformed into the Enrollment Working Group.

Our charter is to develop five-year forecasts – including confidence intervals – for elementary-, middleand high-school enrollment. With the exception of the forecast of the high school, forecasts for individual schools are not within the scope of our responsibilities. In parallel with our work, the Ad hoc Schools Master Planning Committee is responsible for developing the capacity needs of the LPS.

On March 11, 2014, we presented an interim report to the School Committee, which revealed the surprising finding that virtually all of the growth in enrollment from 2003 to 2013 could be attributed to growth in the number of students living in apartments. Over the next several months, we refined our analyses, and on September 10, 2014, we issued a progress report with preliminary forecasts for the three school levels. A summary of the report was presented to the School Committee on September 16, 2014. Since then, we have incorporated the most recent data and have further refined our analyses and forecasts.

In this report we summarize our work, present our five-year forecasts for use by those with the responsibility for planning school facilities, and describe the next activities we plan to undertake to ensure that the analyses we introduce in this report can be carried out by LPS and Town staff.

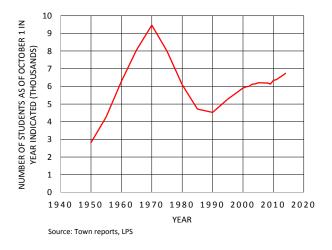
We begin in the following section with a brief history of enrollment in Lexington schools from 1950 to the present in which we describe the impact of evolving demographics on enrollment. After summarizing the data sources used in our work in Section 4, we use those data in the following section to characterize the evolution of Lexington's housing stock, the recent growth in the fraction of the housing stock occupied by families with students and the trend in the number of students residing in housing units with at least one student. These analyses lay the foundation for one of three forecasting methods described in Section 6. We present our five-year forecasts in Section 7. There, we project substantial growth in enrollment at all levels for the next five years, and demonstrate that there exists enough room in Lexington's housing stock to accommodate further growth. For elementary schools, in particular, we forecast that, by the 2019-2020 school year, enrollment could grow by the equivalent of an elementary school if the upper end of the range of our forecast is realized. In Section 8, we assess our forecasts. Finally, in Section 9, we describe our next steps, including plans to develop a deeper

understanding of data that might provide an early warning of changes with the potential to affect enrollment. This document also includes appendices with technical details on the assessment of forecasts, on the calculation of confidence bands for our forecasts, and on the data used in preparing this report.

#### 3 HISTORICAL REVIEW

The challenge of matching school capacity to enrollment is not new. During the post-war years and through the 1960s, baby boomers drove a rapid rise in enrollment (Figure 1). After reaching a peak of just over 9,600 students in 1969, when Lexington had 11 elementary schools, three middle schools and one high school, enrollment dropped precipitously over the next two decades. The response was dramatic: Four elementary schools (Hancock, Parker, Munroe, Adams) were closed over one four-year period between 1978 and 1981; Hastings was closed in 1986. By 1990, when enrollment dropped to just over 4,500, Lexington had only five elementary schools, and had closed one of its middle schools. In the early 1990s, enrollment began to grow again. The re-opening of Hastings School in 1995 raised the number of schools to the current level. More recently, rebuilding, renovations and the addition of modular classrooms have increased the system's capacity. These steps, however, have not kept pace with the rising enrollment.

Figure 1 – Public School Enrollment from 1950 to 2014



What led to the decline in enrollment in the 1970s? We believe it can be traced to two demographic changes. The first is that mothers, who arrived in the 1950s and 1960s and whose children attended Lexington's public schools, passed their childbearing years and continued to live in Lexington. This explanation is supported by the observation that the percentage of households with children under 18 dropped from more than 60 percent in 1960 to just over 30 percent in 1990 (Figure 2). The second demographic change was the decrease in the number of children per family. In 1970, average number of students in a household with children under 18 was 1.91; by 1990, the number had fallen to 1.33.

The growth in enrollment from 1990 to 2010 is primarily the result of a 33% rise in the number of households with children under 18 (Figures 2). In addition, there has been a slight increase in the

number of students in households with children under 18 (Figure 3). The growth may be attributed to the grandchildren of baby-boomers now passing through the school system. It may have been tempting, therefore, to suspect that the plateau in enrollment, beginning in 2005 (Figure 1), signaled the peak of the so-called baby-boom echo, and that enrollment would soon decline. A small drop in enrollment in 2009 appeared to confirm the hypothesis.

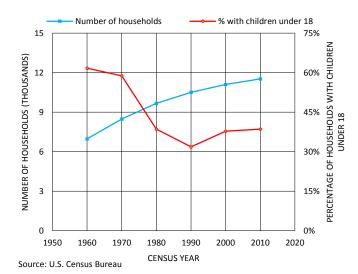


Figure 2 – Households and percentage of households with children under 18

The drop proved to be illusory. Since 2009 enrollment has grown, confounding the forecasts, particularly for the elementary grades (Figure 4). In the worst case, the forecast made in FY2007 (the 2006-2007 school year) underestimated elementary school enrollment in FY2012 by almost 500 students over five years, the equivalent of a school. The FY2010 forecast for FY2014 – four years out – fell short by about 475 students. As we shall see in Section 7, all forecasting methods are challenged by turning points, but not all forecasting methods perform equally poorly at these points.

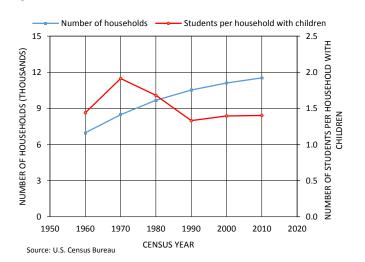


Figure 3 - Households and number of students in households with children under 18

In contrast to the elementary school forecasts, the forecasts for the middle schools (Figure 5) and the high school (Figure 6) have fared well. For example, the FY2010 forecast for middle schools – the forecast with the largest error shown in Figure 3 – underestimated enrollment in FY2015 by fewer than 150 students. The FY2010 forecast for high school enrollment in FY2015 has a slightly higher error: It falls short by 190 students. Because five-year enrollment forecasts for the middle school and high school are based on students already in the system, the forecasts can be expected to be more accurate than those for the elementary grades.

Figure 4 - Elementary-school enrollment: history and LPS projections

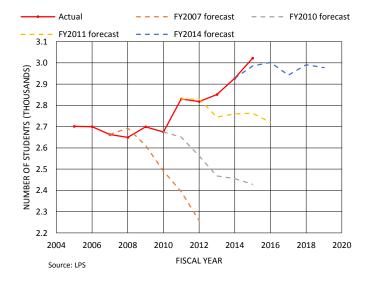
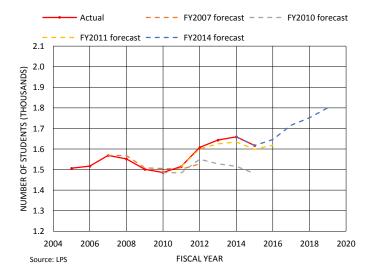


Figure 5 – Middle-school enrollment: History and LPS projections



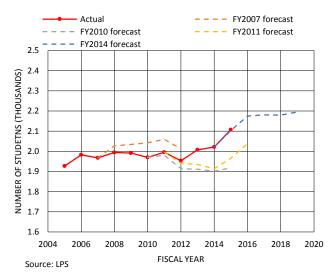


Figure 6 – High-school enrollment: History and LPS projections

The high-level explanations of enrollment since 1950 and the errors associated with forecasts made over the past several years for the elementary grades suggest the need for a forecasting method based on demographic factors. Before considering such a method, we analyze data on Lexington's housing as well as demographic data on Lexington's students. But first, we describe the data used in the analyses.

#### **4** DATA SOURCES

Our analysis of enrollment patterns was made possible by controlled access to a variety of historical municipal and school department data sets. Primary data for our analyses were obtained from the following sources:

- annual October 1 student census reported to the Massachusetts Department of Education by the Lexington Public Schools from FY2004 through FY2015;
- annual Town Census data for FY2004 through FY2015 as maintained by the Town Clerk's office;
- annual Lexington Assessors' property records extracted from the Vision database from FY2007 through FY2014.

While the School Department keeps detailed records of each student, historical home address information is not maintained, only the last known address for each student is retained. Our analyses required understanding where each student resided for each year enrolled in the system. To construct this information student records were correlated with the corresponding Town Census records for each year studied. Assessors' property records were used to establish the class of dwelling for each address determining, for example, if a residence should be classified as an apartment, a condominium, a single-family dwelling, or a multi-family residence.

To minimize the risk of exposure of sensitive student records, one member of the EWG served as the data steward and produced de-identified data sets that replaced personally identifiable information

such as names and addresses with randomly generated unique numbers. This privacy preserving technique allowed EWG members to analyze patterns of student and family movement without disclosing information that would identify the individual students or families.

#### 5 DATA ANALYSIS

In Section 3, we suggested that the decline in enrollment, which began in 1970 could be attributed to the drop in the percentage of housing units occupied by students and a reduction in the average number of students living in a home with children under 18. In this section, we take a close look at trends in a set of related demographic variables, and we show that the trends explain the recent changes in enrollment. We also examine some characteristics of the recent enrollment patterns that present challenges to forecasters.

Our analyses recognize that, in a given year, the number of students residing in Lexington and enrolled in the Lexington Public Schools can be expressed as the product of three variables:

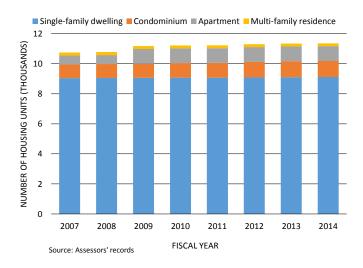
- 1. the number of housing units, ranging from single-family dwellings to individual units in an apartment complex;
- 2. the percentage of these units occupied by students; and
- 3. the average number of students residing in a housing unit with one or more students.

In addition to these students, residing in Lexington, approximately 260 non-residents – primarily under the auspices of the Metropolitan Council for Educational Opportunity, Inc. (METCO) – are currently enrolled in the Lexington Public Schools. Because we wish to forecast the **change** in enrollment, this relatively constant number of non-residents has been omitted from our analyses of growth.

#### 5.1 ANALYSIS OF HOUSING STOCK

Lexington's housing stock falls into four categories: single-family dwellings (approximately 9,100), condominiums (approximately 1,000), apartments (approximately 1,000 units) and multi-family residences (approximately 200) (Figure 7).

Figure 7 - Breakdown of Lexington's housing stock



Although Lexington is often thought of as being fully built out, with new construction generally replacing existing homes, analysis of Assessor's data indicates a steady increase in housing stock. From FY2007 to FY2014, it grew from 10,749 to 11,359 units, an increase of 610 units. Over this period, new apartment units – all stemming from the opening of the Avalon Bay complex between FY2008 and FY2009 – represented about two thirds of the growth. Condominiums added approximately 160 units, while the number of single-family dwellings grew by about 75 units (Figure 8).

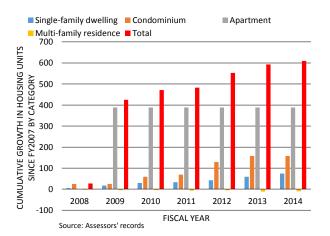


Figure 8 - Cumulative growth of the number of housing units from FY2007 to FY2014

The "condominium" category includes developments normally associated with the designation of "condominium," such as Courtyard Place on Lowell Street and units on Doran Farm Lane, as well as homes that share property and access. At present, there is no formal way of distinguishing between the two sub-categories. Multi-family residences represented the only category showing a slight decline.

#### 5.2 Percentage of housing units occupied by families with students

The percentage of all housing units occupied by students has risen from a low value of 32 percent in FY2010 to a current level of nearly 36 percent. (Figure 9). At present, the percentage is highest in apartments, where it has grown to nearly 44 percent from about 25 percent in FY2004.

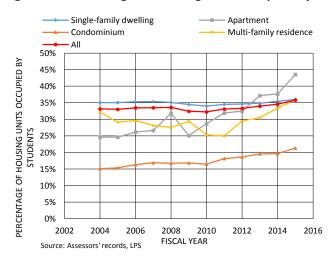


Figure 9 – Percentage of housing units occupied by students

A similar trend has been observed in Brookline, where rising enrollment has been ascribed, in part, to growth in the number of student residing in apartments.<sup>1</sup> For single-family dwellings, the percentage has risen to about 36 percent after dipping to a low value of 34 percent in FY2010. Multi-family residences have shown the greatest recent volatility, with the percentage of units with students now at about 36 percent. While the percentage is rising for condominiums, it remains well below the levels in the other housing categories.

How does the current overall level compare with the percentage attained in 1970? While it is not possible to provide a direct comparison, an approximate answer can be arrived at by considering a related variable, the percentage of households with children under 18. According to the data in **Figure 2**, its 1970 and 2010 values were 59 and 39 percent respectively. By comparison, the percentage of housing units occupied by students in 2010 was 32 percent. Thus, it might be reasonable to estimate that, in 1970, the percentage of housing units occupied by students – rather than children – would have been in the neighborhood of 50 percent – well above its current value. This suggests that there remains substantial headroom for this variable before it approaches the value it was estimated to reach 45 years ago.

#### 5.3 AVERAGE NUMBER OF STUDENTS PER HOUSING UNIT WHERE STUDENTS ARE PRESENT

While Lexington's housing stock and the percentage of housing units occupied by families with students are growing, the average number of students in a housing unit with at least one student has been slowly declining since 2005 (Figure 10). To avoid repetition of the long phrase "average number of students in a housing unit with at least one student" we adopt "student density" as the short-hand designation of this variable. Currently, overall student density has a value of 1.61.

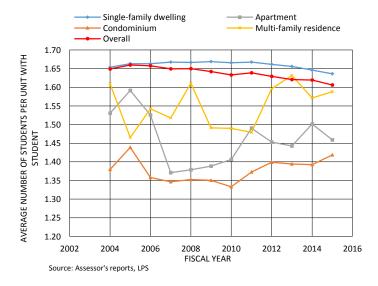


Figure 10 – Average number of students in housing units with at least one student ("student density")

The highest student density is found in single family homes, but the number has been declining slowly for the past four years. By contrast, it has been rising in apartments and condominiums. Since both of

12

<sup>&</sup>lt;sup>1</sup> Final Report. Brookline School Population and Capacity Exploration (B-SPACE) Committee. September 2013. http://www.brooklinema.gov/740/Brookline-School-Population-Capacity-Exp

these housing categories account for about 9 percent of units, the impact of their rise on the overall student density is not as significant as the drop in student density in single-family dwellings. The fluctuation in student density in multi-family residences may be ascribed to dynamic rental patterns and the inevitable volatility associated with the low number of these units, currently about 190.

#### 5.4 ENROLLMENT VOLATILITY

Anecdotes abound regarding the almost-annual surprises in enrollment numbers. The recent history of year-to-year changes in Kindergarten enrollment confirms the stories (Figure 11). As the figure demonstrates, total Kindergarten enrollment has fluctuated almost annually by as much as the equivalent of three Kindergarten classes of 18 students; the largest swings correspond to percentage changes ranging from 12 to 16 percent. Not only do the fluctuations add to forecasting uncertainty, they pose a challenge for planning staff levels. The official registration process for Kindergarten begins when registration packets are mailed out in mid-February for children whose names are recorded via the Town census. The process intensifies after Kindergarten Orientation in March, and continues over the summer months with new move-ins. Kindergarten has been most unpredictable grade for enrollment forecasts.

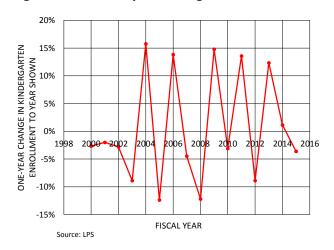


Figure 11 - Volatility in Kindergarten enrollment

#### 5.5 CHURN IN ENROLLMENT

Another aspect of enrollment volatility is the churn, which we define here as the sum of the students arriving from outside LPS in a given year who have never previously been enrolled in the LPS and the number of students who attended a particular school in the previous year but are no longer enrolled there. For elementary schools, the numbers exclude the flow into Kindergarten and the outflow of graduating fifth graders. Rather than using the absolute numbers, we normalize the churn to the average enrollment and express it as a percentage (Figure 12).

The percentages are surprisingly high. In recent years, Bowman and Bridge have had the highest percent churn, with Bowman reaching 20 percent in calendar 2010. Hastings typically exhibits the lowest value – just under 10 percent.

In addition to contributing to the volatility of the enrollment data, the churn presents a challenge to the teachers, who need to contend with a substantial number of students who are new to the school.

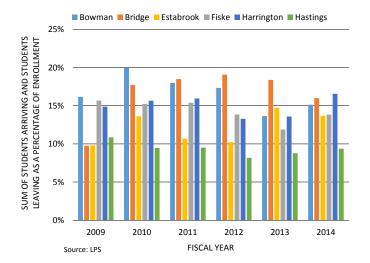


Figure 12 – Churn in elementary schools

#### **6** Forecasting methods

Before presenting our enrollment forecasts in Section 7, we describe three distinct methods along with an assessment of their advantages and shortcomings.

#### 6.1 COHORT SURVIVAL METHOD

The most widely used technique for projecting school enrollment is the Cohort Survival Method (CSM). It is based on a simple concept: Knowing the current number of students in, say, sixth grade, it is assumed that next year the number of students in seventh grade will be approximately the same. In other words, about 100% will "survive." Some students may leave, but they will be replaced by new arrivals. The ratio of the number of students in a given grade to the number in the preceding grade in the previous year is known as the progression rate.

Thus, to project next year's enrollment in a K-to-12 system one multiplies the current enrollment in each grade by the appropriate progression rate. Kindergarten enrollment is traditionally estimated by multiplying births five years earlier by the so-called Birth-to-Kindergarten progression rate. When making projections, it is common practice to average over several years' data to calculate progression rates.

In recent years, the progression rates beyond first grade have ranged from 0.98 to 1.08, with the lower values in the high school. A progression rate greater than one implies a net in-migration – more students arriving than leaving as the cohort advances by one grade. Because most progression rates are greater than 1, the cumulative effect can be significant. For example, in FY2015, the 12<sup>th</sup> grade class is 30% larger than the Kindergarten class of FY2002, when it was nominally launched.

The attractiveness of the CSM lies in the ease of obtaining the data on enrollment and births and on the simplicity of the calculation. Moreover, the method accurately forecasts the movement of enrollment bubbles, cohorts of one or more years of higher-than-average enrollment. Provided that the progression rates are stable – a critical assumption – one can make five-year enrollment projections with some confidence, thereby giving the community time to ensure that appropriate school facilities are in place. The primary drawback of the CSM is that it is not based on any insight or understanding of the underlying demographic variables.

While the recent progression rates required to perform five-year projections of middle-school and high-school enrollment have been relatively stable, making CSM an obvious candidate for forecasting middle-and high-school enrollment, the Birth-to-Kindergarten progression rate in Lexington has been rising and variable. It reached a value of 2.4 in FY2013, up from 1.4 ten years earlier. More significantly, births to Lexington mothers are a poor predictor of Kindergarten enrollment five years later: A regression of Kindergarten enrollment from FY2000 to FY2015 as a function of births five years earlier shows a R<sup>2</sup> value of 0.1. (R<sup>2</sup> is a parameter measuring goodness of fit; it ranges between 0 and 1, with 1 signifying a perfect fit and 0 indicating absence of a fit). Under these circumstances, it is not appropriate to use CSM to make forecasts of elementary school enrollment if the method relies on births and the Birth-to-Kindergarten progression rate. However, the CSM may be appropriate for forecasting elementary school enrollment if Kindergarten enrollment is estimated as the average of recent years' Kindergarten enrollment. When the CSM is based on such a model, we refer to it as the Modified Cohort Survival Method (MCSM).

#### 6.2 Linear Extrapolation of enrollment

Forecasts based on the Linear Extrapolation Method (LEM) use parameters (slope and intercept) derived from a linear regression of several years of enrollment data. In contrast to the CSM, the LEM permits calculation of the quality of the fit using the parameter R<sup>2</sup>. In addition, it permits rigorous calculation of a confidence interval for the forecast (signifying that the forecast values have some probability, say 90 percent, of falling within the interval).

It is important to note, however, that the confidence intervals have significance only with respect to the data and the linear regression. They may not be interpreted to mean that the forecaster has a particular level of confidence in the forecast. Since the underlying assumption of the LEM is that the linear trend will continue, the method is not expected to be reliable in the vicinity of turning points – points where enrollment is peaking or reaching a low point. In any event, in the vicinity of turning points, the R<sup>2</sup> of the regression is likely to be unacceptably low, say <0.5, and the regression will be meaningless.

In contrast to the CSM or MCSM, the LEM is not capable of forecasting enrollment bubbles. However, it shares the drawback of providing no insight into the underlying demographics. It thus provides no rigorous means of allowing the forecaster to make adjustments in light of some anticipated change in the environment, such as the construction of a large apartment complex.

#### 6.3 Housing Demographic Method

The analyses summarized in Sections 5.1 - 5.3 lead naturally to a method based on the three variables – the number of housing units, the percentage of housing units occupied by students and student density. We refer to this as the Housing Demographic Method (HDM), and calculate the forecast by multiplying

individual forecasts for the three variables. The method is applicable to each of the three school levels or to total enrollment. For each school level, one would use the projection of total housing units and a subset for the other two variables.

An attractive feature of the HDM is the ability to create and analyze scenarios based on the variables. For example, instead of relying solely on a forecast for housing units based on a regression, one could insert values based on planned developments expected to produce a substantial increase in housing units. Forecasters could also track home sales, and use the sales rates to adjust the percentage of homes occupied by students: An accelerating sales rate, signaling an increased rate of displacement of households without school-age children by families with students, might lead the forecasters to increase the forecast values of the percentage of units occupied by students.

The HDM shares with the LEM the inability to forecast enrollment bubbles. Also, because the HDM may rely on regressions, it could be unreliable at turning points or during periods where the variables are volatile. However, this would be the case only if the regressions were used without trying to interpret trends in the details of the underlying variables.

#### 7 FIVE-YEAR FORECASTS

Because we wish to develop forecasts for planning school capacity, and because major construction takes several years to plan and execute and is likely to require substantial investment, our objective to develop forecasts with two important characteristics:

- 1. Accuracy Our methods should forecast future enrollment as accurately as possible, while acknowledging that no forecast is perfect.
- 2. *Robustness* The forecasts should not be sensitive to the volatility of the data on which they are based.

#### 7.1 Forecasts for the high school and the middle schools

We decided to base our forecasts for middle- and high-school enrollment on the CSM. (Because five-year forecasts for middle schools using the CSM do not require Kindergarten enrollment, MCSM and CSM give the same results). Our decision to use CSM is based on the relative accuracy and robustness of the method revealed by an assessment of the errors of ex-post forecasts<sup>2</sup> using the CSM and the LEM and on the simplicity of the method. Details of the assessment are provided in **Appendix A**. In addition to comparing the two methods, we compared forecasts using CSM based on three- and five-year averages of progression rates. We found that using progression rates based on the most recent five years yielded lower errors in ex-post forecasts for both the high-school and middle-school enrollment.

In FY2020, enrollment in the High School is projected to reach nearly 2290 students,<sup>3</sup> a growth of 265±120 relative to the FY2014 level **(Figure 13)**. The 90<sup>th</sup> percentile confidence intervals were estimated from the distribution of errors relative to mean errors in ex-post forecasts as explained in

<sup>2</sup> Ex-post forecasts, which use historical data to forecast more recent historical values, are employed to assess how well the method has worked in the past and, by extension, how well it might work in the future.

<sup>&</sup>lt;sup>3</sup> The August 2014 LPS forecast projects a FY2020 high school enrollment of 2265; it does not include confidence intervals. Our forecasts are based on enrollment as of October 1, 2014, the date for official FY2015 reports.

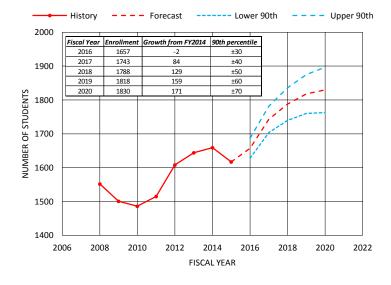
**Appendix B.** We use the term "90<sup>th</sup> percentile" to indicate that, on the basis of the method used to calculate the forecast, there is a 90 percent probability that the forecast value lies between the confidence limits.

History ---- Forecast ----- Lower 90th ---- Upper 90th Fiscal Year | Enrollment | Growth from FY2014 | 90th percentile ±60 ±80 ±100 NUMBER OF STUDENTS FISCAL YEAR

Figure 13 – CSM forecast for high-school enrollment using 5-year averages of progression rates

Using the CSM, we find that, in FY2020, enrollment in the middle schools is forecast to reach  $1830\pm70$  – an increase of about 170 over the FY2014 level (Figure 14). Relative to the magnitude of the enrollment the  $90^{th}$  percentile confidence intervals are narrower than those for the high school. The difference is explained in Appendix B.

Figure 14 - Forecast for middle-school enrollment based on CSM with 5-year averages of progression rates



#### 7.2 Forecast for the elementary schools

For performing five-year forecasts of elementary school enrollment, we have chosen the HDM. Our selection is based on a comparison of ex-post forecasts of the HDM, the MCSM and the LEM described in **Appendix A**. There we demonstrate that, when judiciously used, the HDM is able to forecast turning points.

Before presenting the forecast, we discuss the data and projections of the three variables used in calculating forecasts using the HDM.

**Housing stock.** Since the opening of the Avalon Bay complex in FY2009, housing stock has grown at the rate of about 39 units per year **(Figure 15)**. As shown in the figure, barring any major developments, the number of housing units is projected to continue growing at the rate of about 0.4 percent per year.

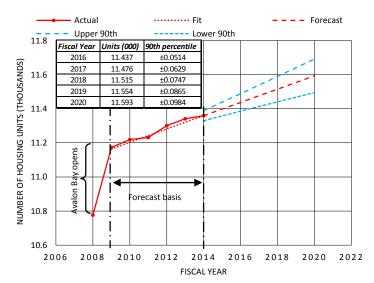
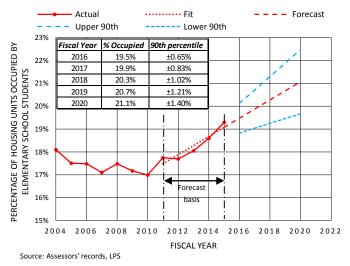


Figure 15 - History and forecast of total housing units

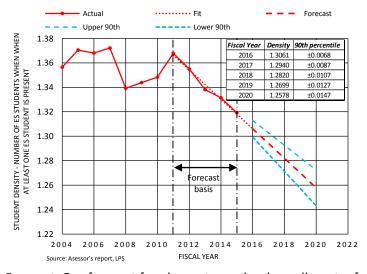
**Percentage of units occupied by elementary school students.** After reaching a minimum of 17 percent in FY2010, possibly a reflection of the 387 units added with the opening of Avalon Bay, the percentage rose to 19.4 percent in FY2015. It is projected to reach about 21 percent in FY2020 with a confidence interval of ±1.4 percent (**Figure 16**). The projection is based on a linear regression of the data from FY2011 to FY2015, and corresponds to an annual increase of about 2 percent in this variable.

Figure 16 - History and forecast of the percentage of housing units occupied by elementary school students



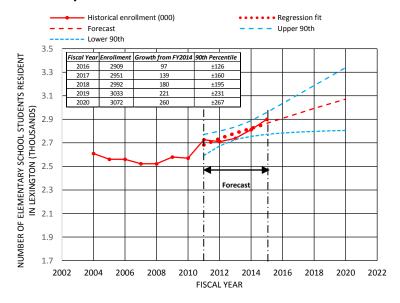
Average number of elementary school students in a housing unit with elementary school students. From FY2011, this variable has been dropping linearly at about 0.01 students per year. It is projected to drop to 1.26 in FY2020 (Figure 17). This represents an annual rate of change of 0.9 percent.

Figure 17 – Average number of elementary-school students in a housing unit where at least one elementary-school student is present (elementary-school student density)



Forecast. Our forecast for elementary school enrollment – for residents only – is the product of the projections of the three variables. In FY2020, the enrollment is projected to have grown by 260±267 since FY2014 (Figure 18). If enrollment turns out to grow at the rate described by the upper confidence limit, the number of additional students would correspond to about one elementary school. The method of calculating the confidence interval is described in **Appendix B**.

Figure 18 - Forecast for elementary-school enrollment (residents only) through FY2020 based on the product of housing units, percentage occupied by elementary-school students and density for elementary-school students



#### 7.3 SUMMARY OF FORECASTS

In **Table 1**, we summarize our forecasts for FY2020. The forecasts differ slightly from the preliminary projections presented to the School Committee in September because we have updated the data to include October 1, 2014 enrollment and because our elementary-school forecasts had been generated with the LEM. In addition, the system-level forecast was expressed as the sum of the individual forecasts.

Table 1 – Summary of enrollment forecasts for FY2020

Grade Group	Method	Enrollment in FY2020	Growth over FY2014
Elementary (K-5)	HDM	3188 ± 267	260 ± 267
Middle School (6-8)	CSM	1830 ± 70	171 ± 70
High School (9-12)	CSM	2290 ± 120	269 ± 120
Total System	HDM	7279 ± 410	671 ± 410

The difference between the forecasts of total system enrollment (7279) and its growth (671) and the sum of the projections for enrollment (7308) and growth (700) for the individual Grade Groups is the result of performing a separate forecast for the total system. In performing the total system forecast, we used the HDM. Because the forecasts for the elementary schools and the total system consider only students residing in Lexington, we adjusted the FY2020 forecasts by adding in the number of FY2014 non-resident students in each category to arrive at the final projection. The adjustment accounts for the difference between the elementary-school enrollment in FY2020 (3072) shown in **Figure 18** and the projected enrollment shown in **Table 1** (3188).

#### 8 Discussion

In introducing the five-year forecasts we proposed two characteristics for judging our projections: accuracy and robustness.

Some support for the robustness of the forecasts for the middle schools and the high school, both based on CSM, comes from the near agreement between forecasts based on three- and five-year averages of the progression rates. Had the forecasts differed substantially, we would have had to conclude that our projections are sensitive to selections of the historical data.

An assessment of the likely accuracy of our forecasts for the middle schools and the high school may be drawn from the results of the ex-post forecasts described in **Appendix A**. Simply put, the ex-post forecast are quite accurate. This suggests that our five-year forecasts are likely to prove accurate — when the confidence bands are taken into account.

It is more difficult to make a categorical assessment of the accuracy and robustness of the HDM-based forecast for the elementary grades. However, as demonstrated in **Appendix A**, it is the only method that successfully forecast the turning point in the FY2008 to FY2009 interval. Thus we believe that it could also be used to detect the inevitable point at which elementary-school enrollment stops growing or begins to decline, most likely as a result of changes in the percentage of housing units occupied by students.

What, then, is a reasonable upper limit to the fraction of housing units occupied by students? A five-year linear extrapolation to FY2020, shown in **Figure 9**, indicates that it would rise to about 37 percent, well short of levels attained in the 1970s. On the other hand, the percentage of apartment units occupied by students is rising much more rapidly. A five-year linear extrapolation indicates that in FY2020 56% of apartments will be occupied by families with students. Could this grow to 70% or 75%? Although it is not possible to answer the question, there remains – for now – ample headroom. However, concerns regarding headroom could prove moot: Rising prices for single-family homes and condominiums as well as rising rental rates could dampen interest in Lexington's schools and curtail enrollment growth before the percentage of units occupied by students reaches a limit. Building a model to resolve this issue would appear to be a daunting task.

In summary, our forecasts, based on two distinct methods, project a five-year increase in elementary-school enrollment that ranges from a slight decrease – which we deem unlikely – to a growth relative to FY2014 levels that is the equivalent a typical elementary school. For the middle schools and the high school we also forecast substantial increases in enrollment relative to FY2014: 170 and 265 students respectively. Finally, in the next five to ten years, we do not expect Lexington to run out of housing stock for parents wishing to move here for the sake of our schools.

In generating our forecasts we have not attempted to incorporate fundamental drivers of enrollment in Lexington such as MCAS scores, housing costs in Lexington compared to other communities in the Boston metropolitan area, national and regional trends in family size, or the attractiveness of public schools relative to private schools. Building a model that takes into account all likely variables might give the appearance of rigor, but credible forecasts of the explanatory variables do not exist.

#### 9 RECOMMENDED NEXT STEPS

We recommend analyses of turnover rates and in- and out-migration to measure the extent of net inmigration of families with school-age children. Such an undertaking would require data on families who rent apartments or homes. At present such information is available only in conjunction with home or condominium sales.

In addition, responsibility for performing enrollment forecasts based on a combination of Lexington Public School data and Town data needs to be turned over to appropriate staff. As the transfer is being initiated, we recommend that the designated staff members work with the EWG in documenting the current process, particularly the acquisition and management of the required data.

#### **10** ACKNOWLEDGMENTS

We are grateful to: Aaron Henry, Assistant Planning Director, who provided us with housing data; the development staff at AvalonBay Communities for a detailed breakdown of apartment units in the three Avalon properties in Lexington; Judy Baldasaro, David Bordenca and Dorinda Goodman of the Lexington IT department who recovered historical property assessment data for our use; Donna Hooper, Town Clerk, and her staff for recovering periodic extracts of the Town's census data; Maureen Kavanaugh, Director of Planning and Assessment, LPS for discussion regarding the forecasting process; Alan Levine, who demonstrated how to forecast Kindergarten enrollment in the CSM by basing it on past Kindergarten levels; Peter Beebee for reviewing the methods for calculating confidence intervals; Ruth Quinn-Berdell for acquainting us with the processes used by the LPS to forecast enrollment and for providing historical data; and to the School Committee for stimulating us to develop a method for estimating confidence intervals for CSM forecasts. Finally, we would like to thank Dr. Paul Ash for his support, encouragement, suggestions and patience over the past 12 months.

#### **APPENDICES**

#### APPENDIX A - EX-POST FORECASTS

A method of judging the accuracy and reliability of a forecasting method is to assess how it would have performed in the past. This is accomplished by using a subset of historical data to forecast more recent history. If these so-called ex-post forecast are applied to a period that has not witnessed major changes, it may not be possible to distinguish effectively between alternative methods because the test would not be sufficiently demanding. However, if the past includes turning points – reversals in the rate of change of enrollment, such as we have witnessed over the past 15 years or so – then such periods create the opportunity for an effective test.

Ex-post forecasts for middle-school and high-school enrollment using CSM and LEM

Past forecast for middle-school enrollment prepared by the Lexington Public Schools, shown in **Figure 5**, suggest that the CSM yielded very good agreement between actual and forecast values, and suggested that the CSM should continue to be used for these projections. Nonetheless, we wished to compare the ex-post performance of the CSM and the LEM for middle-school forecasts (**Figure A1 and A2**). Although we calculated ex-post forecasts from FY2005 to FY2014, for the sake of simplicity we show only four forecasts. We selected FY2005, FY2010 and FY2012 forecasts because of their large forecast errors compared to other years. Comparison of the results shown in the figures leads to the conclusion that the CSM yields lower errors. (The R² values shown in **Figure A2** represent the goodness of fit to the five years of data used to perform the linear regressions, not the fit to the forecast to subsequent enrollments. Note that there appears to be no correlation between the goodness of fit and the forecast errors).

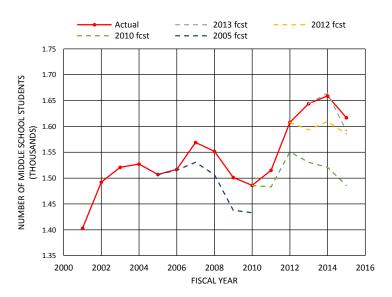


Figure A1 - Ex-post forecast for MS using CSM based 5-year averages of progression rates

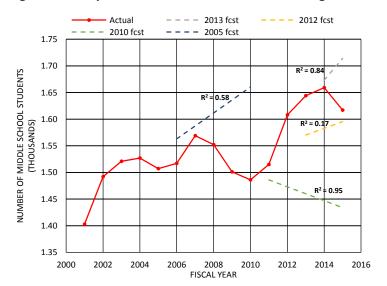


Figure A2 - Ex-post forecast for middle schools using LEM with a 5-year forecast basis

On the basis of this comparison and because of the simplicity of the CSM and its capability of forecasting enrollment bubbles, we decided to use the CSM for middle school forecasts.

We also compared forecast errors for the CSM (using progression rates averaged over the most recent five years) and the LEM (five-year forecast basis) for high-school enrollment. On the basis of the comparison, we decided to use the CSM.

#### Ex-post forecasts for elementary-school enrollment using MCSM and LEM

To select the appropriate method for forecasting elementary-school enrollment we began by comparing the forecast errors of the MCSM and the LEM. For the former we used a five-year average of the most recent Kindergarten enrollment and of progression rates; for the latter we employed a linear regression of five years (Figures A3 and A4). We found that, when the projections do not span a turning point, both methods yield forecasts with acceptable differences between forecast and actual values five years after the date of the forecast. But both methods fail when they project through a turning point, as evidenced by forecasts made in FY2008 and FY2009. Table A1 compares the forecast and actual values for the two methods.

Both methods under-forecast as one might expect when a rise in enrollment follows a decline. However, the MCSM performs slightly better. We would also expect both methods to over-forecast when enrollment flattens out or begins to decline, with the MCSM showing slightly less bias. On the basis of these observations, we prefer the MCSM, based on five-year averages for Kindergarten enrollment and progression rates, to the LEM.

Figure A3 - - Ex-post forecasts for the elementary schools using MCSM based on 5-year averages

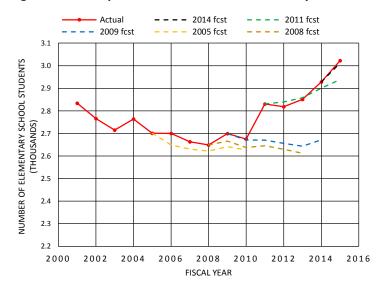


Figure A4 - Ex-post forecast for the elementary schools based on LEM with 5-yr forecast basis

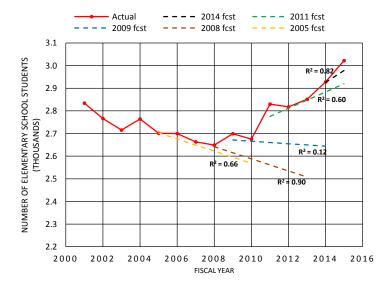


Table A1 – Difference between forecast and actual values of elementary school enrollment for several forecast dates for MCSM, based on five-year average of Kindergarten enrollment and progression rates, and LEM using a five-year base period

Fiscal date of	Difference between fore 5 years after	Comments	
forecast	MCSM	LEM	
2005	-44	-107	
2008	-246	-343	$R^2 = 0.90 \text{ for LEM}$
2009	-256	-282	$R^2 = 0.12$ for LEM
2011	-83	-103	4 years after forecast
2014	-11	-43	1 year after forecast

#### Ex-post assessment of the HDM

Could the HDM have been able to forecast the growth in elementary school enrollment that started in FY2008? If the answer is yes, then it may also be capable of anticipating a decline in enrollment following the current period of sustained growth – a vital capability. In **Table A2**, we demonstrate that, with reasonable assumptions, forecasts using the HDM in FY2008 would indeed have anticipated the period of growth that started in FY2009.

The "History" section of **Table A2** presents historical data on the three variables (number of housing units, the percentage of these occupied by elementary schools students and elementary-school student density) for the dominant categories of housing. From FY2004 to FY2008, it shows total enrollment (of elementary-school students residing in Lexington) dropping by about 60. Analysis reveals that declining enrollment of students in single family homes, caused by a 4% decline in the percentage of single family homes occupied by families with elementary school student and a slight drop in density, was offset by growth in the number students in apartments and condominiums. For these categories, density also dropped slightly, but the percentage of housing units occupied by students was growing.

Table A2 – Ex-post forecasts, made in FY2008 and partly based on a reasonable assumptions, for FY2009 and 2011.

			History	,		Estin	nate	
Fiscal year	2004	2005	2006	2007	2008	2009	2011	Assumptions and comments
Single-family dwellings	9043	9043	9043	9043	9048	9048	9048	For 2009 and 2011 use 2008 values
% single-family dwellings occupied by ES students	19.1%	18.4%	18.4%	18.1%	18.3%	18.3%	18.3%	For 2009 and 2011 use 2008 values
ES density in single-family dwellings	1.351	1.372	1.369	1.379	1.344	1.344	1.344	For 2009 and 2011 use 2008 values
Number of ES students in single-family dwellings	2334	2281	2280	2254	2221	2221	2221	
Apartment units	647	647	647	647	647	1034	1034	Add 387 to account for Avalon Bay opening in FY2009
% apartment occupied by ES students	11.6%	13.0%	13.9%	13.9%	16.2%	15.0%	18.0%	Assume 15% in 2009; 18% in 2011
ES density in apartments	1.360	1.345	1.256	1.256	1.276	1.276	1.276	For 2009 and 2011 use 2008 values
Number of ES students in apartments	102	113	113	113	134	198	238	
Condominiums	908	908	908	908	933	933	933	For 2009 and 2011 use 2008 values
% condominiums occupied by ES students	6.7%	7.7%	7.4%	7.2%	8.6%	8.6%	8.6%	For 2009 and 2011 use 2008 values
ES density in condominiums	1.230	1.229	1.239	1.246	1.150	1.150	1.150	For 2009 and 2011 use 2008 values
Number of ES students in condominiums	75	86	83	81	92	92	92	
Total number of ES students	2511	2480	2476	2448	2447	2511	2551	Growth from 2008: in 2009 = 64; in 2011 = 104

In the "Estimate" columns, we present forecasts for FY2009 and FY2011 based on historical data and an anticipated addition to the housing stock. In FY2008 it was known that Avalon Bay, comprising 387 apartment units would open the following year. Thus, given the observed interest in apartments, it would have been reasonable to anticipate that the new apartments would be occupied by families with students. If we assume that in FY2009 and FY2011, 15 percent and 18 percent, respectively, of apartment units would be occupied by elementary-school students, and that all the other variables would remain at their FY2008 levels, then we conclude that, from FY2008 to FY2009, enrollment in the elementary grades would grow by about 65 students. By FY2011, enrollment would grow by an additional 40 students. Although the growth anticipated by these hypothetical forecasts is modest when compared to the actual growth from FY2009 to FY2011, the HDM is the only method that appears

\_

<sup>&</sup>lt;sup>4</sup> The actual values in FY2009 and 2011 were 16.6 and 19.1 percent, respectively.

capable of at least partially forecasting turning points – provided that careful thought is given to trends in the underlying variables.

#### APPENDIX B - ESTIMATION OF CONFIDENCE INTERVALS

Our charter calls for the development of confidence intervals for all forecasts because we believe that responsible planning of facilities requires awareness of the uncertainty associated with the forecasts.

Before presenting the calculation of confidence intervals for our forecasting methods, it is important to draw attention to the distinction between statistical fit, e.g., a high value of R², and a model's validity or reliability: A forecast based on an excellent fit to the data can turn out to be inaccurate because of a structural change in the environment. All confidence intervals formally calculated suffer from this problem, including the ones whose calculations we present below. Our challenge is compounded by the fact that the two forecasting methods that we have employed do not lend themselves to standard, formal approaches for calculating confidence intervals.

#### Estimation of confidence interval for the MCSM

We developed a pragmatic method to estimate the confidence interval for MCSM based on the forecast errors generated in the ex-post forecasts from FY2005 to FY2014. For each ex-post forecast, we recorded the errors for each year of the forecast and then calculated the standard deviation of the errors for that year. **Table B1** demonstrates our method for total enrollment. For a one-year horizon, the forecast errors range from -154.6 in the FY2010 forecast to +21.8 in the FY2011 forecast. The standard deviation of these values is 49.1. Next, we assumed that the errors had a normal or bell-curve distribution, and computed the 90 percent confidence boundaries as ±1.65 x standard deviation. Finally, we adjusted the values to arrive at "smoothed 90% confidence limits." The final steps are shown in **Table B2**.

Table B1- Initial steps in estimation of confidence intervals for the MCSM

Fcst horizon	FY2014 fcst	FY2013 fcst	FY2012 fcst	FY2011 fcst	FY2010 fcst	FY2009 fcst	FY2008 fcst	FY2007 fcst	FY2006 fcst	FY2005 fcst	Std dev
1 year	-10.3	-35.6	-5.2	21.8	-154.6	-3.8	-32.7	14.9	15.5	-51.1	49.1
2 years		-59.9	-61.5	6.9	-161.9	-159.6	-37.4	-14.2	24.6	-32.7	62.4
3 years			-132.1	-27.7	-192.7	-162.0	-184.7	-17.8	-4.2	-27.1	76.5
4 years				-82.9	-259.3	-207.9	-188.1	-165.5	4.3	-57.9	86.7
5 years					-337.9	-256.1	-238.9	-154.0	-121.4	-48.2	95.5

Table B2 - Final steps in estimating confidence interval for the MCSM

Forecast	Standard	90th	Smoothed
horizon	deviation	percentile	90% CL
1 year	49.1	±81.1	±80
2 years	62.4	±103.0	±100
3 years	76.5	±126.2	±120
4 years	86.7	±141.0	±140
5 years	95.5	±157.5	±160

We would like to note that error estimates like these rely on statistics that work well if one has a process such as manufacturing where errors are additive and thus the total system errors tend to follow a standard normal bell curve, one has hundreds of measurements so one can accurately estimate the relevant statistics, and the underlying process is not shifting. Unfortunate those conditions do not hold here. What we have done is show how well the forecast methods did over one particularly short and challenging period of time. While not rigorous, these error estimates provide at least a notional sense of the degree of uncertainty. It is worth noting that these error estimates are approximately the same magnitude as the error from the growth continuing for an extra year or stopping a year early, adding an additional consistency check. While these estimates are not precise, they provide a good sense of the magnitude of the expected uncertainty.

#### Estimation of confidence intervals for the HDM

The HDM consists of the product of projections of the three variables – number of housing units (H), percentage of housing units occupied by students (P) and student density (D). For each variable, confidence limits can be calculated as shown in standard textbooks. If we express the forecast, including the confidence limits, as the product  $(H\pm\Delta H)(P\pm\Delta P)(D\pm\Delta D)$ , then the forecast value is HPD, and the confidence limits are approximately given by  $\pm (HP\Delta D + PD\Delta H + DH\Delta P)$ .

#### Appendix C - Data used to construct the forecasts and most of the figures

#### Historical values of enrollment and births (LPS data)

FY	Births	К	1	2	3	4	5	6	7	8	9	10	11	12	K-5	6-8	9-12	Sys Total
1981	Direns	209	257	282	307	375	400	444	424	483	409	562	620	667	1830	1351	2258	5439
1982		211	229	267	281	320	378	406	517	499	576	521	548	629	1686	1422	2274	5382
1983		211	243	245	270	288	332	374	477	531	461	580	537	558	1589	1382	2136	5107
1984		267	293	293	267	322	334	370	451	482	513	474	563	550	1776	1303	2100	5179
1985		271	306	297	297	272	320	349	371	447	472	506	467	577	1763	1167	2022	4952
1986		259	316	321	301	312	275	318	347	376	449	462	515	462	1784	1041	1888	4713
1987		273	301	315	330	315	312	281	326	346	362	437	456	510	1846	953	1765	4564
1988		297	337	315	343	349	316	322	285	330	334	351	405	452	1957	937	1542	4436
1989		304	324	340	334	346	352	325	327	293	330	337	345	413	2000	945	1425	4370
1990		341	347	326	350	335	358	353	335	340	283	332	336	357	2057	1028	1308	4393
1991		354	379	352	340	364	335	366	369	348	351	277	335	332	2124	1083	1295	4502
1992		377	379	384	340	352	369	345	376	372	347	330	301	316	2201	1093	1294	4588
1993		364	428	387	396	347	354	380	347	376	368	337	342	280	2276	1103	1327	4706
1994		379	408	438	420	417	356	383	388	344	377	352	346	321	2418	1115	1396	4929
1995	287	377	439	415	452	417	420	385	394	387	337	377	342	336	2520	1166	1392	5078
1996	280	429	429	450	435	455	425	427	400	394	386	341	358	349	2623	1221	1434	5278
1997	283	412	486	457	462	439	471	427	438	411	361	378	334	371	2727	1276	1444	5447
1998	294	381	455	521	462	466	439	470	456	446	396	385	374	319	2724	1372	1474	5570
1999	264	413	437	465	540	471	468	454	470	456	423	399	360	361	2794	1380	1543	5717
2000	243	402	467	455	470	547	476	474	452	466	441	423	386	348	2817	1392	1598	5807
2001	247	394	467	482	461	479	551	484	469	450	457	428	407	370	2834	1403	1662	5899
2002	235	383	462	488	494	452	487	540	481	471	442	455	422	400	2766	1492	1719	5977
2003	208	349	447	464	504	496	455	492	547	482	468	441	443	414	2715	1521	1766	6002
2004	226	404	413	446	478	515	508	471	509	547	482	469	442	439	2764	1527	1832	6123
2005	220	354	450	429	468	482	518	504	482	521	543	475	465	444	2701	1507	1927	6135
2006	205	403	413	464	446	472	502	530	513	474	522	534	475	451	2700	1517	1982	6199
2007	181	385	433	437	489	446	473	517	535	517	473	516	515	463	2663	1569	1967	6199
2008	213	338	444	459	455	496	457	495	521	536	500	473	512	509	2649	1552	1994	6195
2009	200	388	400	455	478	470	508	465	509	527	527	495	460	509	2699	1501	1991	6191
2010	212	376	413	440	465	494	487	519	458	509	508	507	497	458	2675	1486	1970	6131
2011	187	427	461	449	464	502	527	513	530	472	484	514	505	492	2830	1515	1995	6340
2012	216	389	483	482	470	493	501	547	515	546	465	479	507	502	2818	1608	1953	6379
2013	219	437	424	513	507	464	506	536	580	528	542	483	473	509	2851	1644	2007	6502
2014		442	485	460	530	524	487	531	553	575	522	535	482	482	2928	1659	2021	6608
2015		426	486	515	484	555	556	511	537	569	563	518	530	496	3022	1617	2107	6746

#### Data on housing by category (highlighted columns contain estimated data as shown to right of the table)

		7 9019 9031 9043 9048 9060 9072 9076 9085 9102 9117 9129 12 1 850 879 908 933 933 967 977 1037 1066 1066 1095 29 7 597 597 597 597 985 985 985 985 985 985 985 985 0												
Fiscal year	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	Forecast Delta (FY2004-FY2006, FY2015)	
Single-family dwelling	9007	9019	9031	9043	9048	9060	9072	9076	9085	9102	9117	9129	12	
Condominium	821	850	879	908	933	933	967	977	1037	1066	1066	1095	29	
Apartment	597	597	597	597	597	985	985	985	985	985	985	985	0	
Multi-family unit	205	203	201	199	196	194	194	192	193	187	189	187	-2	
Other	2	2	2	2	2	1	1	1	1	1	2	2	0	
Total	10632	10671	10710	10749	10776	11173	11219	11231	11301	11341	11359	11398	39	

#### Percentage of housing units occupied by students (system-wide)

_	_						•						
		Percentage occupied by all students											
Fiscal year	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	
Single-family dwelling	35.2%	35.1%	35.3%	35.3%	35.1%	34.4%	34.0%	34.5%	34.6%	34.8%	35.5%	36.0%	
Condominium	16.7%	16.4%	16.8%	16.9%	16.7%	16.8%	16.4%	18.1%	18.6%	19.5%	19.6%	20.7%	
Apartment	24.6%	24.6%	26.1%	26.6%	31.8%	25.1%	28.7%	31.9%	32.5%	37.2%	37.7%	43.6%	
Multi-family unit	31.2%	28.6%	29.4%	28.1%	27.6%	29.4%	25.3%	25.0%	29.5%	30.5%	33.3%	36.4%	
Overall percentage	33.5%	33.3%	33.6%	33.5%	33.6%	32.4%	32.2%	33.1%	33.3%	34.0%	34.6%	35.7%	

#### Percentage of housing units by elementary-school students

		Percentage occupied by elementary school students												
Fiscal year	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015		
Single-family dwelling	19.2%	18.4%	18.4%	18.1%	18.3%	18.2%	17.7%	18.1%	17.9%	18.0%	18.5%	19.0%		
Condominium	7.4%	8.2%	7.6%	7.2%	8.6%	8.8%	8.7%	9.5%	10.7%	10.7%	11.3%	11.4%		
Apartment	12.6%	14.1%	15.1%	15.1%	17.6%	14.8%	17.5%	20.3%	20.0%	23.8%	24.3%	26.5%		
Multi-family unit	18.0%	16.3%	14.9%	15.1%	11.7%	10.8%	11.3%	13.0%	15.0%	14.4%	15.3%	21.4%		
Overall percentage	18.1%	17.5%	17.5%	17.1%	17.5%	17.2%	17.0%	17.7%	17.7%	18.0%	18.6%	19.3%		

# Average number of students in a housing unit when at least one student is present – System-wide student density

		Density for all students												
Fiscal year	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015		
Single-family dwelling	1.654	1.663	1.663	1.667	1.667	1.669	1.666	1.668	1.661	1.656	1.646	1.636		
Condominium	1.380	1.439	1.358	1.346	1.353	1.350	1.333	1.373	1.399	1.394	1.392	1.419		
Apartment	1.531	1.592	1.526	1.371	1.379	1.389	1.406	1.490	1.453	1.443	1.501	1.459		
Multi-family unit	1.609	1.466	1.542	1.518	1.611	1.491	1.490	1.479	1.596	1.632	1.571	1.588		
Overall density	1.649	1.660	1.658	1.649	1.650	1.642	1.633	1.639	1.629	1.621	1.620	1.606		

# Average number of elementary-school students in a housing unit when at least one elementary school student is present – Elementary-school student density

	Density for elementary school students											
Fiscal year	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
Single-family dwelling	1.351	1.372	1.369	1.379	1.344	1.357	1.366	1.376	1.371	1.360	1.343	1.338
Condominium	1.230	1.229	1.239	1.246	1.150	1.146	1.202	1.333	1.297	1.272	1.192	1.160
Apartment	1.360	1.345	1.256	1.256	1.276	1.192	1.192	1.290	1.254	1.192	1.285	1.253
Multi-family unit	1.297	1.242	1.300	1.333	1.478	1.524	1.318	1.240	1.310	1.407	1.379	1.250
Overall density	1.357	1.370	1.368	1.372	1.339	1.344	1.348	1.368	1.355	1.338	1.331	1.319